

# Open membranes, ribbons and deformed Schild strings

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We analyze open membranes immersed in a magnetic three-form field-strength  $C$ . While cylindrical membranes in the absence of  $C$  behave like tensionless strings, when the  $C$  flux is present the strings polarize into thin membrane ribbons, locally orthogonal to the momentum density, thus providing the strings with an effective tension. The effective dynamics of the ribbons can be described by a simple deformation of the Schild action for null strings. Interactions become non-local due to the polarization, and lead to a deformation of the string field theory, whereby string vertices receive a phase factor proportional to the volume swept out by the ribbons. In a particular limit, this reduces to the non-commutative loop space found previously.

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Open strings in large magnetic backgrounds at low energy are best described as excitations of a non-commutative Yang-Mills theory. While interesting questions remain unanswered as to their precise dynamics and observables, non-commutative gauge theories are well defined field theories, albeit with an infinite series of higher derivative interactions. The Moyal star product entering their Lagrangian is a simple consequence of the nature of the microscopic degrees of freedom, which behave like non-relativistic elastic dipoles in a strong magnetic field [1].

In contrast, open membranes, whether in non-trivial 3-form backgrounds or in vacuo, are much more elusive objects. They are believed to be the appropriate degrees of freedom of the 5-brane on which they are required to end, yet the precise way in which the self-dual dynamics of the two-form gauge potential (and other members of the (2,0)-supersymmetric multiplet on the 5-brane) arises by quantization of the open membrane is completely mysterious. This is not to mention of course the case of a stack of  $N$  5-branes, for which the two-form analogue of non-Abelian Yang-Mills is not known. Indeed, there are even reasons to doubt that a field theory description may be appropriate, since membranes ending on two coinciding 5-branes presumably behave like tensionless strings, with an infinite tower of states and no tunable coupling (see however [2] for recent progress).

Nevertheless, there are reasons to believe that membranes in magnetic backgrounds should be more tractable. For one, we do not expect that switching on a magnetic or electric background will change the spectrum: the (2,0) multiplet should still describe open membranes ending on a single 5-brane with a  $C$  field. Second, in a large 3-form field strength the membrane dynamics

should be dominated by the boundary coupling

$$\int_{\partial M_2} C_{\mu\nu\rho} X^\mu dX^\nu \wedge dX^\rho \quad (1)$$

which should be easier to quantize than the Nambu-Goto part of the membrane action: indeed, in the string theory case, quantizing the first-order quantum mechanics on the boundary of the open string is the most direct path to the Moyal product. Steps in this direction have been reviewed in [3], and have lead to a heuristic proposal for the leading deformation of Abelian two-form dynamics, compatible with invariance under volume-preserving diffeomorphisms [3].

The purpose of this paper is to pursue the analysis of open membranes in a large magnetic  $C$  field at a purely classical level, building on earlier work [3, 4].

We first observe that the relevant kinematical degrees of freedom are membranes with two boundaries, and cylindrical topology, which in the absence of  $C$  would behave as tensionless strings. Our main finding is that the magnetic 3-form polarizes these strings into thin ribbons, orthogonal to the local momentum density. The “open membrane theta parameter”, first introduced by indirect methods in [5], is now physically interpreted as the polarizability of these ribbons. In this process, the originally tensionless strings gain inertia and can be described as non-relativistic tensionful strings, albeit with a non-standard worldsheet action. Just as with electric dipoles in a magnetic field, this polarization induces non-local interactions, which can be summarized by a deformation of the closed string field theory, analogous to the non-commutative deformation. In a particular gauge, we recover the non-commutative string found in [6, 7].

A possible concern with this line of reasoning is the fact, exploited in the OM proposal [17], that a magnetic

three-form field strength,  $H_{123}$ , cannot be large unless the dual electric component  $H_{045}$  approaches the critical electric field value. While membranes with a single boundary in this limit tend to grow in the (045) directions until they break, cylindrical membranes remain tensionless in the (045) directions. The turning on of the  $H_{045}$  components therefore will not qualitatively affect our conclusions.

Finally, it would be useful to make contact with more formal approaches such as [14, 15, 16], which have focussed on the deformation of the  $L_\infty$  algebra underlying the closed string field theory, in the small  $C$  limit.

### The non-commutative string, revisited

As a starting point, let us recall some basic features of the dynamics of an open string in a magnetic field as originally described in [1] with some additional details to aid the generalisation to the membrane. In the process, we shall gain understanding of the open string “metric” and “non-commutativity parameter” which we will be able to extend to the membrane case.

Open strings end on D-branes. When only one end is immersed in a magnetic field, the string behaves like a charged particle and being trapped on Larmor orbit, takes little part in the transport properties of the system. In contrast, when both of the ends are immersed in the same magnetic field, the string is globally neutral and behaves like an electric dipole. When moving at a velocity  $\vec{v}$ , the magnetic Lorentz force  $\vec{F}_l = \pm e\vec{v} \wedge \vec{B}$  exerted at the end must cancel the elastic force  $\vec{F}_e = \pm k\vec{\Delta}$ , where  $k$  is the elasticity constant, leading to a polarisation  $\Delta = e\vec{v} \wedge \vec{B}/k$  transverse to the direction of motion. A useful analogy is that of vortices in two-dimensional fluid dynamics: by the Magnus force, two vortices of opposite vorticity are able to propagate forward, the velocity field of one carrying the other along.

To see how this non-relativistic description emerges from the usual relativistic fundamental string, recall that the electromagnetic coupling imposes the boundary condition

$$\partial_\sigma X^i + B_{ij} \partial_\tau X^j = 0, \quad \text{at } \sigma = 0, \pi. \quad (2)$$

This can be solved along with the bulk equation of motion  $(\partial_\sigma^2 - \partial_\tau^2)X^\mu = 0$  to give the zero-mode solution,

$$X^i = p_0^i \tau + B_{ij} p_0^j \sigma \quad (3)$$

(we set the string tension to 1, and assume that the target space metric is that of Minkowski space; indices are raised or lowered with the Kroneker symbol  $\delta_{ij}$ ). From this expression, it is apparent that the string is stretched into a dipole of length

$$\Delta_i = B_{ij} p_0^j. \quad (4)$$

The canonical linear momentum is related to  $p_0$  by

$$P^i = (\partial_\tau X^i - B_{ij} \partial_\sigma X^j) = (1 + B^2) p_0^i \quad (5)$$

so indeed the dipole is stretched proportionally to its momentum  $P^i$ . This gives an elongation

$$\Delta^i = \Theta^{ij} P_j, \quad \Theta = \frac{B}{1 + B^2}. \quad (6)$$

The “open string non-commutativity parameter”  $\Theta^{ij}$ , introduced in [13], can thus be viewed as the dynamic polarizability of the open string dipole in a magnetic field. As argued in [1], the fact that open string dipoles interact via their end points implies non-local interactions in the effective field theory, e.g. for two point interactions,

$$\mathcal{L} \sim \int d^n x \phi_1 \left( x + \frac{1}{2} \Theta^{ij} P_j \right) \phi_2 \left( x - \frac{1}{2} \Theta^{ij} P_j \right) \quad (7)$$

$$= \int d^n x \phi_1 * \phi_2 \quad (8)$$

which is precisely the effect of the Moyal star product. More generally,  $n$ -point vertex are pick up a phase proportional to the area of the polygon formed by the incoming dipoles.

To see how the “open string metric” arises, let us compute the total energy of the string. Using that  $X^0 = \alpha' E \tau$ , and that the worldsheet Hamiltonian,

$$H = \int (\partial_\tau X)^2 + (\partial_\sigma X)^2 = (1 + B^2) p_0^2 - E^2 + N \quad (9)$$

should vanish on physical states ( $N$  is the contribution of the excited levels of the string) the energy is given by the following dispersion relation

$$E = \sqrt{m^2 + G^{ij} P_i P_j}, \quad G_{ij} = (G^{ij})^{-1} = (1 + B^2) \delta^{ij} \quad (10)$$

where  $m^2 = N$ . The effective metric  $G_{ij}$  governing the dependence of the energy on the momentum is the “open string metric” as discussed in [13].

One can now check that the balance between the Lorentz and tensile forces is satisfied. The velocity is  $v^i = p_0^i/E$ , hence the Lorentz force is given by:

$$F_l^i = B_{ij} v^j = \frac{1}{E} \Delta^i. \quad (11)$$

On the other hand, expressing the momentum  $P$  as a function of the elongation  $\Delta$  into the energy (10), one derives

$$F_t^i = \frac{\partial E}{\partial \Delta^i} = \frac{1 + B^2}{B^2} \frac{\Delta^i}{E}. \quad (12)$$

This indeed cancels the Lorentz force  $F_l$  in the limit of large magnetic field  $B$ , validating the assumption that the string is in its ground state.

It is instructive to redo this computation without fixing the conformal gauge (as this is not available in the membrane case). The equations of motion and boundary conditions for  $X^\mu$  read

$$\partial_\alpha (\sqrt{\gamma} \gamma^{\alpha\beta} \partial_\beta X^\mu) = 0 \quad (13)$$

$$\sqrt{\gamma} \gamma^{\sigma\sigma} \partial_\sigma X^i + B_{ij} \partial_\tau X^j = 0 \quad \text{at } \sigma = 0, \pi \quad (14)$$

where  $\gamma_{ab}$  has to be equal to the induced metric  $\partial_\alpha X^\mu \partial_\beta X_\mu$ , up to a conformal factor. The zero-mode ansatz

$$X^i = p_0^i \tau + \Delta^i \sigma, \quad \gamma_{\alpha\beta} = \text{diag}(m^2, \Delta^2) \quad (15)$$

where  $m^2 = E^2 - p_0^2$ , is thus a solution if

$$\frac{\Delta^i}{|\Delta|} = B_{ij} \frac{p_0^j}{m}. \quad (16)$$

While this equation does not specify the length of  $\vec{\Delta}$ , upon substituting  $p_0$  by the physical momentum

$$P_i = \sqrt{\gamma} \gamma^{\tau\tau} \partial_\tau X^i - B_{ij} \partial_\sigma X^j = \frac{|\Delta|}{m} (1 + B^2) p_0^i \quad (17)$$

it becomes equivalent to the relation (6) above.

### From strings to ribbons

We now come to the case of non-commutative membranes in a large  $C_{123}$  magnetic flux. Our first observation is that, in order to contribute to the transport properties of the (2,0) theory, the membrane should have at least two boundaries on the 5-brane with non-vanishing flux. This is because, as noticed in [3], for a single boundary the equations of motion of the boundary string

$$C_{ijk} \partial_\sigma X^j \wedge \partial_\tau X^k = 0 \quad (18)$$

imply that  $X^i$  has to depend on the worldsheet coordinates through one function  $f(\tau, \sigma)$  only: the boundary of the membrane therefore spans a static string in the  $(X^1, X^2, X^3)$  plane. A useful analogy is that of a vortex line in a three-dimensional fluid: just as in two-dimensions, the transverse motion of a vortex line is effectively confined by the rotational motion of the fluid itself, on Landau-like orbits. In addition, there exist soft modes propagating along the vortex line known as Kelvin modes. In fact, as noticed in [3, 4], the boundary coupling (1) is precisely the one describing the Magnus effect in fluid hydrodynamics. Of course, in contrast to the two-dimensional case, the total vorticity is not conserved and a vortex line may slowly shrink and disappear, just as the membrane boundary may shrink to a point and leave the five-brane, under the effect of its tension.

On the other hand, membranes with two boundaries have no overall charge and therefore can propagate freely: this is the analogue of configurations of vortex anti-vortex lines in hydrodynamics. In the absence of a  $C$  field (and with no Higgs vev), the two boundaries lie on top of each other, leading to an effectively tensionless string, the tentative fundamental degrees of freedom of the (2,0) theory. In the presence of a magnetic field however, it is easy to see that these tensionless strings polarize into thin ribbons, whose width is proportional to the local momentum

density. Indeed, the canonical momentum on the membrane, neglecting the contribution of the Nambu-Goto term, is

$$P^i = C_{ijk} \partial_\sigma X^j \partial_\rho X^k \quad (19)$$

where  $\sigma$  is the coordinate along the boundary string, and  $\rho$  the coordinate normal to it. The ribbon thus grows as

$$\Delta^i \sim \partial_\rho X^i = \frac{1}{C |\partial_\sigma X|^2} \epsilon_{ijk} P^j \partial_\sigma X^k \quad (20)$$

where we retain in  $\Delta$  only the component orthogonal to  $\sigma$  (the parallel component could be reabsorbed by a diffeomorphism on the membrane worldvolume).

In order to study more precisely this polarization, let us consider a simple classical solution corresponding to an infinite strip of width  $\Delta$  moving at a constant velocity  $v$  transverse to it: We thus consider the classical solution

$$X^i = p_0^i \tau + u^i \sigma + \Delta^i \rho. \quad (21)$$

The boundary condition

$$\sqrt{\gamma} \gamma^{\rho\rho} \partial_\rho X^i - C_{ijk} \partial_\sigma X^j \partial_\tau X^k = 0 \quad (22)$$

with induced metric  $\gamma = \text{diag}(m^2, |u|^2, |\Delta|^2)$ , implies that the direction of the polarization vector is orthogonal to the plane formed by the tangent vector to the string  $\vec{u} = \partial_\sigma \vec{X}$  and the local velocity  $\vec{p}_0 = \partial_\tau \vec{X}$ ,

$$\frac{\vec{\Delta}}{|\Delta|} = C \frac{\vec{u}}{|u|} \wedge \frac{\vec{p}_0}{m}. \quad (23)$$

Calculating the local canonical momentum

$$P^i = \sqrt{\gamma} \gamma^{\tau\tau} \partial_\tau X^i - C_{ijk} \partial_\sigma X^j \partial_\rho X^k \quad (24)$$

$$= \frac{|\vec{u}| |\vec{\Delta}|}{m(1 + C^2)} \left[ (1 + C^2) p_0^i - C^2 \frac{\vec{u} \cdot \vec{p}_0}{|\vec{u}|^2} u^i \right] \quad (25)$$

one may express the local velocity in terms of  $P^i$ ,

$$p_0^i = \frac{m}{|\vec{u}| |\vec{\Delta}| (1 + C^2)} \left[ P_i + C^2 \frac{\vec{u} \cdot \vec{P}}{|\vec{u}|^2} u^i \right] \quad (26)$$

and obtain the relationship between the membrane polarization and canonical momentum,

$$\vec{\Delta} = \Theta \frac{\vec{u}}{|u|^2} \wedge \vec{P}, \quad \Theta = \frac{C}{1 + C^2}. \quad (27)$$

For convenience, we will use the gauge  $m = |\vec{u}| |\vec{\Delta}|$  from now on.

We thus recover the “open membrane non-commutativity parameter”  $\Theta$ , defined in [5]. In this work, the “open membrane non-commutativity parameter” and “open metric” were determined by studying the physics of five-branes probing supergravity duals with  $C$ -flux longitudinal to the probe brane world volume. We now understand this result classically as the the polarizability of open membranes in a  $C$ -field. We shall return to the issue of the “open membrane metric” shortly.

### An effective Schild action for string ribbons

Membranes are notoriously difficult to quantize. Since the effect of the magnetic background is to polarize the boundary strings, hence given a non-zero tension, it is interesting to ask if one can write down an effective string theory that may be more tractable. For this, let us start with the light-cone formulation [18] of the membrane, with Hamiltonian

$$P^- = \int d\sigma d\rho \frac{1}{2P^+} [(p_0^i)^2 + g] \quad (28)$$

where  $g$  is the determinant of the spatial metric, hence the square of the area element (and the membrane tension is set to 1). In this gauge, one should enforce the constraint

$$\partial_\sigma X^i \partial_\rho \partial_\tau X^i - \partial_\rho X^i \partial_\sigma \partial_\tau X^i = 0 \quad (29)$$

which is trivially satisfied on zero-mode configurations (21). For a thin ribbon of width  $\vec{\Delta}$  given by (27), the square of the area element is

$$g = |\vec{u} \wedge \vec{\Delta}|^2 = \frac{C^2}{(1+C^2)^2} \left[ \vec{P}^2 - \frac{(\vec{u} \cdot \vec{P})^2}{|u|^2} \right] \quad (30)$$

On the other hand, using (26), the kinetic energy may be written as

$$(\vec{p}_0)^2 = \frac{1}{(1+C^2)^2} \left[ \vec{P}^2 + C^2(C^2+2) \frac{(\vec{u} \cdot \vec{P})^2}{|u|^2} \right] \quad (31)$$

The total Hamiltonian thus takes the form

$$P^- = \int d\sigma \frac{1}{2P^+(1+C^2)} \left[ P^2 + C^2 \frac{(\vec{P} \cdot \partial_\sigma \vec{X})^2}{|\partial_\sigma X^i|^2} \right] \quad (32)$$

From this expression, specifying to a gauge choice where  $\vec{P}$  and  $\partial_\sigma \vec{X}$  are orthogonal, we see that the effective metric in the transverse directions is rescaled by a factor of  $(1+C^2)$ ,

$$G_{ij} = [1+C^2] \delta_{ij} \quad (33)$$

This agrees with the membrane metric found from very different considerations in [5, 10], up to the conformal factor  $Z = (1 - \sqrt{1 - 1/K^2})^{1/3}$  with  $K = \sqrt{1+C^2}$ . It should however be noted that the evidence for this conformal factor is rather indirect, and its non-analyticity  $Z \sim 1 - |C|/3 + O(C^2)$  at weak  $C$  is not understood. It is also possible that quantum corrections on the membrane world-volume may correct our classical result.

Finally, we may perform a Legendre transform on  $P_i$  to find the Lagrangian density of the ribbon,

$$\mathcal{L} = \int d\sigma \frac{1}{2} (\partial_\tau X^i)^2 + \frac{C^2}{2|\partial_\sigma X^i|^2} \sum_{i,j} \{X^i, X^j\}^2 \quad (34)$$

where we have defined the Poisson bracket on the Lorentzian string worldsheet[23] as  $\{A, B\} = \partial_\sigma A \partial_\tau B - \partial_\sigma B \partial_\tau A$ . Note that the relative sign between the two terms in (34) is consistent with the fact that they both contribute to kinetic energy. For vanishing  $C$ , (34) reduces to the Lagrangian for a tensionless string, as expected. While we have mostly worked at the level of zero-modes, it is easy to see that (34) remains correct for arbitrary profiles  $X^i(\tau, \sigma)$ , as long as the dependence on membrane coordinate  $\rho$  is fixed by Eqs. (21), (27).

After fixing the invariance of the Lagrangian (34) under general reparameterizations of  $\sigma$  by choosing  $|\partial_\sigma X^i| = 1$ , we recognize in the second term the Schild action, which provides (in the case of a Lorentzian target-space) a unified description of both tensile and tensionless strings, depending on the chosen value for the conserved quantity  $\omega = \{X^i, X^j\}^2$  [20]. This term dominates over the first in the limit of large  $C$  field. For any finite value however,  $\omega$  is not conserved, and the second term in (34) can be interpreted as the action for a non-relativistic string with tension proportional to  $C$ .

As usual, it is possible to give a regularization of this membrane action, by replacing the Poisson bracket (now in light-cone directions on the worldsheet) by commutators in a large  $N$  matrix model. One thus obtains a lower-dimensional analogue of the type IIB IKKT matrix model [21],

$$P^- = \frac{1}{2P^+} \left( [A_0, X^i]^2 + C^2 \sum_{i < j} [X^i, X^j]^2 \right). \quad (35)$$

It would be interesting to understand how the matrix regularization distinguishes the Lorentzian worldsheet from more usual Euclidean one. We leave the study of this model and its supersymmetric version for future work.

### Non-commutative string field theory

Just like open strings, open membranes interact only when their ends coincide. Since their boundaries are tensionless closed strings which polarize into thin ribbons in the presence of a strong  $C$  field, one may expect that the effect of the  $C$  field can be encoded by a deformation of the string field theory describing the membranes boundaries. Despite the fact that string field theory of closed strings, not to mention tensionless ones, is a rather ill-defined subject, it is natural to represent the string field as a functional in the space of loops. The effect of the polarization of the ribbons can thus be represented by

$$V \sim \int [DX(i)] \Phi \left[ X^i - \frac{1}{2} \frac{\Theta}{|\partial_\sigma X|^2} \epsilon_{ijk} \partial_\sigma X^j \frac{\delta}{\delta X^k} \right] \times \Phi \left[ X^i + \frac{1}{2} \frac{\Theta}{|\partial_\sigma X|^2} \epsilon_{ijk} \partial_\sigma X^j \frac{\delta}{\delta X^k} \right] \quad (36)$$

where we represented the momentum density  $P_i$ , canonically conjugate to  $X^i(\sigma)$ , as a derivative operator in the space of loops. Defining the operators

$$\tilde{X}^i(\sigma) = X^i - \frac{1}{2} \frac{\Theta}{|\partial_\sigma X|^2} \epsilon_{ijk} \partial_\sigma X^j \frac{\delta}{\delta X^k} \quad (37)$$

it is easy to reproduce the non-commutative loop space in the “static” gauge  $X^3(\tau, \sigma) = \sigma$ ,

$$[\tilde{X}^1(\sigma), \tilde{X}^2(\sigma')] = \Theta \delta(\sigma - \sigma') \quad (38)$$

as proposed in [6, 7]. The fact that the transverse fluctuations of a vortex line are effectively confined by an harmonic potential is well known in fluid dynamics. In the more covariant gauge  $|\partial_\sigma \tilde{X}| = 1$ , one obtains a tensionless limit of the  $SU(2)$  current algebra,

$$[\tilde{X}^i(\sigma), \tilde{X}^j(\sigma')] = \Theta \epsilon_{ijk} \partial_\sigma X^k \delta(\sigma - \sigma'). \quad (39)$$

The same relations may be directly obtained by Dirac quantization of the topological open membrane Lagrangian (1).

More generally, much as in the non-commutative case, this deformation amounts to multiplying the closed string scattering amplitudes by a phase factor proportional to the volume enclosed by the ribbons as they interact. It would be very interesting to derive the deformation of the (2, 0) effective field theory from (36), and possibly verify the proposal in [3] motivated by the invariance under volume preserving diffeomorphisms.

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- [1] D. Bigatti and L. Susskind, “Magnetic fields, branes and noncommutative geometry,” *Phys. Rev. D* **62** (2000) 066004 [arXiv:hep-th/9908056].
- [2] P. Arvidsson, E. Flink and M. Henningson, *JHEP* **0306** (2003) 039 [arXiv:hep-th/0306145].
- [3] B. Pioline, “Comments on the topological open membrane,” *Phys. Rev. D* **66** (2002) 025010 [arXiv:hep-th/0201257].
- [4] Y. Matsuo and Y. Shibusa, “Volume-preserving diffeomorphism and noncommutative branes,” *JHEP* **0102**, 006 (2001) [hep-th/0010040].
- [5] D. S. Berman, M. Cederwall, U. Gran, H. Larsson, M. Nielsen, B. E. W. Nilsson and P. Sundell, “Deformation independent open brane metrics and generalized theta parameters,” *JHEP* **0202** (2002) 012 [arXiv:hep-th/0109107].
- [6] E. Bergshoeff, D. S. Berman, J. P. van der Schaar and P. Sundell, “A noncommutative M-theory five-brane,” *Nucl. Phys. B* **590** (2000) 173 [arXiv:hep-th/0005026].
- [7] S. Kawamoto and N. Sasakura, “Open membranes in a constant C-field background and non-commutative boundary” *JHEP* **0007** (2000) 014 [arXiv:hep-th/0005123].
- [8] R. Banerjee, B. Chakraborty and K. Kumar, “Membrane and noncommutativity,” *Nucl. Phys. B* **668** (2003) 179 [arXiv:hep-th/0306122].
- [9] G. W. Gibbons and P. C. West, “The metric and strong coupling limit of the M5-brane,” *J. Math. Phys.* **42** (2001) 3188 [arXiv:hep-th/0011149].
- [10] J. P. Van der Schaar, “The reduced open membrane metric,” *JHEP* **0108** (2001) 048 [arXiv:hep-th/0106046].
- [11] F. Lund and T. Regge, “Unified Approach To Strings And Vortices With Soliton Solutions,” *Phys. Rev. D* **14** (1976) 1524.
- [12] M. Rasetti and T. Regge, “Quantum Vortices And Diff (R3),”
- [13] N. Seiberg and E. Witten, “String theory and non-commutative geometry,” *JHEP* **9909**, 032 (1999) [arXiv:hep-th/9908142].
- [14] J. Park, “Topological open p-branes,” hep-th/0012141; C. M. Hofman and W. K. Ma, “Deformations of Closed Strings and Topological Open Membranes,” hep-th/0102201.
- [15] C. Hofman and J. S. Park, “BV quantization of topological open membranes,” arXiv:hep-th/0209214.
- [16] C. M. Hofman and W. K. Ma, “Deformations of closed strings and topological open membranes,” *JHEP* **0106**, 033 (2001) [arXiv:hep-th/0102201].
- [17] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, “OM theory in diverse dimensions,” *JHEP* **0008**, 008 (2000) [arXiv:hep-th/0006062]. E. Bergshoeff, D. S. Berman, J. P. van der Schaar and P. Sundell, “Critical fields on the M5-brane and noncommutative open strings,” *Phys. Lett. B* **492** (2000) 193 [arXiv:hep-th/0006112].
- [18] B. de Wit, J. Hoppe and H. Nicolai, “On The Quantum Mechanics Of Supermembranes,” *Nucl. Phys. B* **305** (1988) 545.
- [19] R. Dijkgraaf, E. Verlinde and H. Verlinde, “Matrix string theory,” *Nucl. Phys. B* **500** (1997) 43 [arXiv:hep-th/9703030]. L. Motl, “Proposals on nonperturbative superstring interactions,” arXiv:hep-th/9701025.
- [20] A. Schild, “Classical Null Strings,” *Phys. Rev. D* **16** (1977) 1722.
- [21] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, “A large-N reduced model as superstring,” *Nucl. Phys. B* **498**, 467 (1997) [arXiv:hep-th/9612115].
- [22] M. Cederwall and H. Larsson, “M5-branes and matrix theory,” arXiv:hep-th/0312303.
- [23] This should not be confused with the Poisson bracket formulation of the membrane, which refers to the two spatial directions of the membrane world-volume.